Public goods and private gifts

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Abstract

We consider contributions to public goods where contributors receive private benefits based on the amount of their contribution. This turns out to be equivalent to Andreoni's warm glow model, and has interesting properties in the case of a discrete public good.

Those who raise funds for a public purpose often offer private inducements to contributors such as gifts that depend on the level of contribution a contributor makes. This is a way of tying a contribution to a public good and a private good together.

Public TV. Public TV fundraisers often offer private gifts, such as DVDs of special performances as an inducement to contribute. There may be a range of gifts that depend on the amount contributed. A common variation to have a public contribution rewarded by an invitation to a private party or "charity ball." The term "gift" is slightly misleading, as the gift should really be viewed as a form of compensation for contributing to the public good.

Lighthouses. In a well-known paper, Coase [1974] describe the provision of lighthouses in England. Typically, customs officials collected lighthouse fees from ships who docked at ports where their journeys benefited from the lighthouse services. In this case, one could view the total payment as a contribution to the provision of the public lighthouse, with the dock usage as a private "gift." (See Varian [1994] for further discussion of Coase's lighthouse paper.)

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Kickstarter. Kickstarter is an online fundraising mechanism that solicits contributions from donors for creative projects. If total donor contributions exceed a threshold amount, the project is undertaken. If the total donor contribution does not reach the threshold, the donor contributions are refunded. In many cases, the project is licensed under Creative Commons terms and may be freely accessed. However, are also individual gifts for the donors that depend on their contribution level such as signed copies of books, DVDs of performances, T-shirts, and the like.

Crowdfunding and micropatronage. These terms refer to variations on the Kickstarter model. Generally crowdfunding refers to financing a startup of some sort while micropatronage refers to financing a non-profit. However, the terminology is not standardized.

The defining characteristic of these mechanisms is the tying of a public good and a private good. For this to work, the cost of the private good the consumer receives must be small relative to the contribution. Examples such as autographs, DVDs, parties, and so on satisfy this requirement. Below we will assume that the cost of providing the private good is zero.

As the examples illustrate tying a public and private good together is a very common mechanism for funding public goods, but it has received relatively little analysis in the economics literature.

1 Conceptual framework

Let g_i be the amount of the contribution by person i and G be the total amount of some public good provided. The utility of the individual depends on the total amount of the public good provided, G, the gift he receives in exchange for his contribution to the public good, g_i, and his private consumption x_i, and so can be written as $U_i(x_i, G, g_i)$.

This is, of course, the same formulation as the Andreoni [1990] model of impure altruism. In this case, the "impurity" is not a "warm glow" but an explicit benefit from a private good.

2 Provision point mechanisms

In the examples given above, the payoff from the provision of the public good is discrete (it is either provided or not) and the gift to the donor is discrete (they either give enough to get the gift or they don't.) So instead of a "warm glow," due to the gift, we might consider this a "hot or not" model.

One nice feature of the discrete model of public good provision is that an efficient outcome may be obtained as a Nash equilibrium of a "provision point" mechanism. Consider a simple two-person case where the public good can be provided if $g_1 + g_2 \geq 10$, and the value of the public good to each agent is 7. Then a contribution of $g_1 = g_2 = 5$ is a Nash equilibrium, since if either party defects, the good is not provided. Of course, this equilibrium is far from unique as $(6, 4)$ is also an equilibrium as is $(0, 0)$.

Provision point models have been examined by many researchers, including Bagnoli and Lipman [1989],Bagnoli and McKee [1991],Bagnoli et al. [1992], Bagnoli and Lipman [1992],Chadsby and Maynes [1999],Hagel and Roth [1997], Marks and Croson [1998],Marks and Croson [1999],Murphy et al. [2005],Prince et al. [1992],Rondeau et al. [1999], and Rose et al. [2002].

In this model it is never an equilibrium for total contributions to exceed the minimum amount to provide the public good. However, in the fundraising mechanism described earlier, it is common to see the funding threshold exceeded. Could this be due to the private value aspect of these models? This is the question we will examine in the following sections.

3 A discrete public good model with private gifts

Here is the notation we use:

$$\begin{aligned}
u_i &= \text{value of public good to agent } i && (1)\\
r_i &= \text{value of private good to agent } i && (2)\\
g_i &= \text{contribution of agent } i && (3)\\
G &= g_1 + g_2 = \text{total contributions} && (4)\\
\bar{G} &= \text{threshold for total contributions to public good} && (5)\\
\bar{g}_i &= \text{threshold for contribution to receive private good} && (6)\\
& && (7)
\end{aligned}$$

We look at the case of 2 agents. The payoff to agent 2 is given by

$$v_2 = \begin{cases} u_2 + r_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 \geq \bar{g}_2 \\ u_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 < \bar{g}_2 \\ 0 & \text{if } g_1 + g_2 < \bar{G} \end{cases} \tag{8}$$

To avoid trivial cases, we make

Assumption 1. It is efficient to jointly fund the public good, but it is not efficient for an individual agent to fund the public good. $u_1 + u_2 > \bar{G}$ but $u_1 < \bar{G}$ and $u_2 < \bar{G}$.

3.1 Case 1. No private good

We first examine the classic case where there is no private good so $r_i = 0$. Let us define the "indirect utility" for agent 2 as a function of agent 1's contribution, which we write as $v_2(g_1)$.

If $g_1 + g_2 > \bar{G}$, either agent could increase utility by lowering its contribution, so we can rewrite the inequality as an equality. Hence, agent 2 contributes at all, it will choose to contribute $g_2 = \bar{G} - g_1$.

Hence

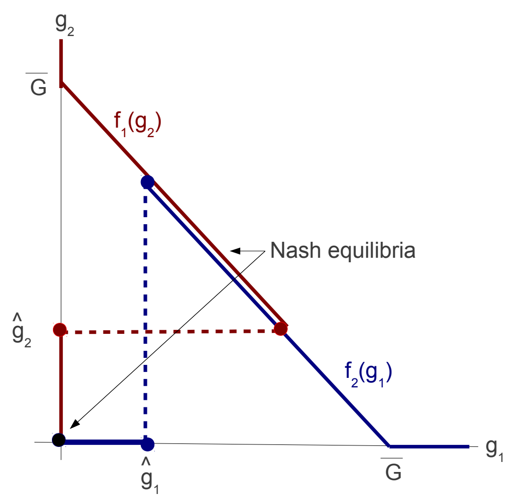
$$v_2(g_1) = \max\{u_2 - \bar{G} + g_1, 0\} \tag{9}$$

Agent 2 is just willing to contribute when $u_2 - \bar{G} + g_1 = 0$. Let $\hat{g}_1 = \bar{G} - u_2$ be the solution to this equation. For $g_1 > \hat{g}_1$, agent 2's optimal choice is to contribute $g_2 = \bar{G} - g_1$, and below $\hat{g}_1$ agent 2 contributes nothing. This gives us agent 2's reaction function:

$$g_2(g_1) = \begin{cases} \bar{G} - g_1 & \text{if } g_1 \geq \hat{g}_1 = \bar{G} - u_2 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The reaction functions for the two agents are shown in Figure 3.1. We see that there is a whole range of equilibria described by the following inequalities.

$$g_1 + g_2 = \bar{G} \tag{11}$$
$$g_1 \geq \bar{G} - u_2 = \hat{g}_1 \tag{12}$$
$$g_2 \geq \bar{G} - u_1 = \hat{g}_2 \tag{13}$$



3.2 Case 2. With private gift

We now examine the case where there is gift that has private value to the contribution. The payoffs to agent 2 are defined above in equation 8. Now agent 2 has three choices: to contribute $g_2 = 0$, to contribute just enough to fund the public good, $g_2 = \bar{G} - g_1$, or to contribute enough to fund the public good and meet the threshold for receiving the private gift.
Threshold for private gift is small. $u_i + r_i > \bar{g}_i$ for $i = 1, 2$. This ensures that *if* the public good is funded, then each agent thinks is it worth paying the threshold amount to receive the private gift.

The point at which agent 2 just begins to give is now given by:

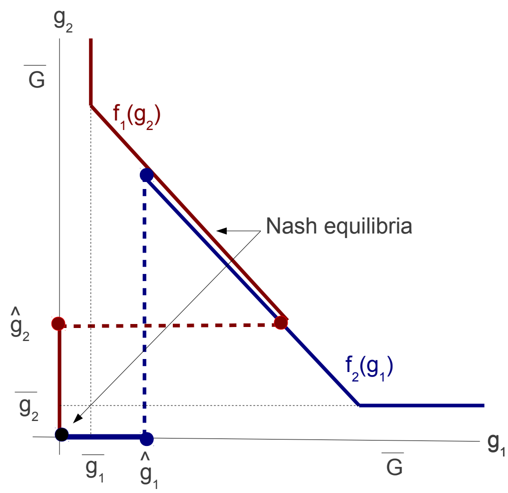
$$u_2 + r_2 + g_1 = \bar{G},$$

so

$$\hat{g}_1 = \bar{G} - u_2 - r_2.$$

At $\hat{g}_1$ agent 2 contributes $g_2(\hat{g}_1) = u_2 + r_2$ which, by assumption, is sufficient to trigger the private gift.

At $g_1 > \hat{g}_1$, agent 2 will contribute the minimum amount necessary to fund the public good and receive the private gift. The amount necessary to fund the public good is $\bar{G} - g_2$ and the amount necessary to receive the



private good is $\bar{g}_2$. Hence the reaction function is given by

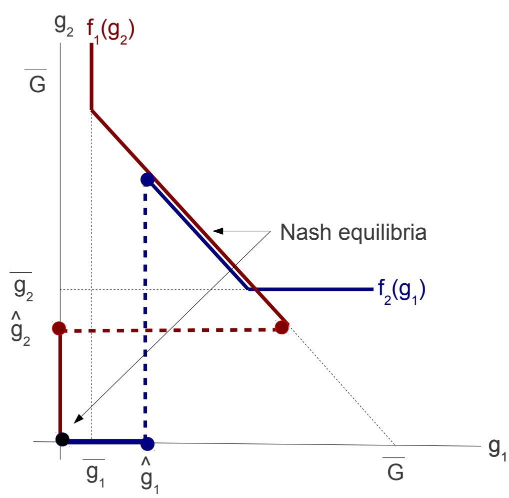
$$g_2(g_1) = \begin{cases} 0 & \text{if } g_1 < \hat{g}_1 \\ \bar{G} - g_1 & \text{if } \bar{G} - \bar{g}_2 \geq g_1 \geq \hat{g}_1 \\ \bar{g}_2 & \text{if } g_1 > \bar{G} - \bar{g}_2 \end{cases} \tag{14}$$

Note that the range of equilibria is larger since the threshold contributions $(\hat{g}_1, \hat{g}_2)$ are larger, due to the extra value provided by the private good. Furthermore, the reaction curve for agent 2 becomes flat at $\bar{g}_2$ when the contribution of agent 1 becomes sufficiently large. Even though the public good would be provided if agent 2 made only a small contribution, it would not be enough to exceed the threshold for receiving the private gift.

There are now three sorts of equilibria. 3.2, we have a range of equilibria, as in the simple case. Figure 3.2 shows a "partially private equilibrium" where $g_1 + \bar{g}_2 = \bar{G}$ and Figure 3.2 shows a "purely private equilibrium" where $\bar{g}_1 + \bar{g}_2 > \bar{G}$.

Note that adding the private good makes the range of equilibria larger on the left-hand side, but smaller on the right-hand side.

If there are many players, the private good thresholds may help reduce the size of the public goods problem in the sense that the "public" part of the funding is relatively smaller. Mathematically, the amount of the purely public good to fund is $G - \sum_{i=1}^{n} \bar{g}_i$, which could be relatively small if the private valuations are large.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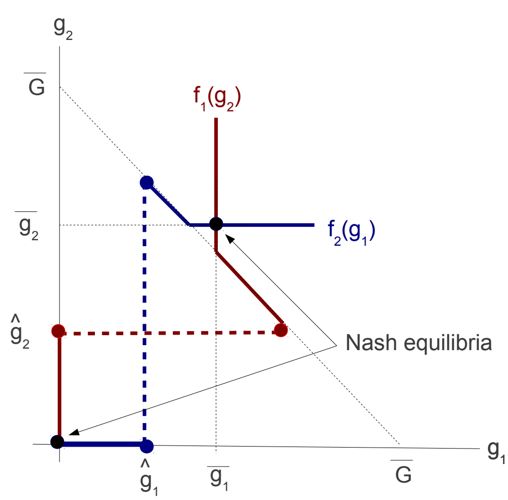
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