Revealed Preference and its Applications

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Abstract

This note contains a short history of my work on the foundations of revealed preference, emphasizing the intellectual influence of Sydney Afriat. I also present a few "novel" applications of revealed preference analysis drawn from some areas of applied economics.

1 Introduction

I was led to revealed preference through a somewhat circuitous route. It started with what Samuelson [1974] called the "money metric utility function" which can be defined as the minimum expenditure necessary to purchase a consumption bundle at least as good as a given consumption bundle x at some fixed set of prices. This concept is very helpful in applied welfare economics and shows up in public finance and industrial organization among other places.

I wondered how one might construct bounds on the money metric utility function using revealed preference techniques. I was familiar with revealed preference as a theoretical construct, but I had no idea how to use it it

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practice. I chatted with Andreu Mas-Colell about this one time when I was visiting Berkeley and he suggested that I read Diewert [1973], which in turn led me to the work of Sydney Afriat.

The first work I probably read was Afriat [1967]. I realized early on that this approach was what I was looking for since it was focused on the computational question rather than the theoretical issues of existence, uniqueness and so on that comprised a large part of the revealed preference literature at that time.

It was difficult to find some of Afriat's publications so I wrote to him and he was kind enough to send me a package of his work in this area. I found the little monograph Afriat [1976] to be the most useful description of his work for my purposes; I'm sure I never would have found it without Sydney's effort.

Once I had access to Afriat's theoretical insights, I was able to answer my original question, at least in part. In Varian [1982] I constructed a tight lower bound to the money metric utility and conjectured a tight upper bound. This upper bound was eventually proved correct by Knoblauch [1992].

Along the way, I realized that there were many other applications of computational revealed preference theory as pioneered by Afriat. I see the applications as falling in four areas.

Consistency. Is a set of observed choices consistent with the utility maximization model? The revealed preference literature, and particularly Afriat's work, answered this clearly, so the only remaining issue was to develop the computer programs to actually check the revealed preference condition, which I did in Varian [1982].

Form. Can the observations be rationalized by a utility function of a given form, such as a separable, homogeneous, quasilinear, and so on? Afriat [1981], Diewert and Parkan [1985], Varian [1983] and several others contributed to this literature.

Recoverability. How can we recover preferences consistent with a set of observed choices? This was my original question, and was answered inVarian [1982] and Knoblauch [1992].

Extrapolation. How can we forecast demand at prices other than those that have been observed? This is a natural extension of the recoverability question and was solved in Varian [1982].

Subsequently, Blundell et al. [2003, 2008] have carried this research program much further along by estimating mixed parametric and non-parametric models. I've tried to summarize some of this work in Varian [2005] but given the rapid progress in this area, this paper is by now somewhat out of date.

2 Welfare effect of price discrimination

In the rest of this paper, I will describe some novel applications of utility theory and revealed preference analysis that arose in other work I have done. The first question involves the welfare effect of price discrimination.

This is a classic question, first raised by Robinson [1933]. Several researchers subsequently looked at the question, such as Schmalensee [1981] but without a lot of progress. In the early 1980s I realized that one could use the properties of the money metric utility function to answer this question.

The setup involves n goods $\mathbf{x} = x_1, \ldots, x_n$ which sell at prices $\mathbf{p} = p_1, \ldots, p_n$. Preferences are summarized by an aggregate quasilinear utility function which can be represented by an indirect money metric utility function of the form $v(\mathbf{p}) + m$, where m is the total amount of money income.

The price vector changes from $\mathbf{p}^0$ to $\mathbf{p}^1$ which results in a demand change $\Delta\mathbf{x} = \mathbf{x}^1 - \mathbf{x}^0$. Let Δc denote the total change in the costs of production, so that the change in welfare is given by $\Delta W = \Delta v - \Delta c$.

Fact 1 *The change in welfare satisfies the following bounds:*

$$\mathbf{p}^0\Delta\mathbf{x} - \Delta c \geq \Delta W \geq \mathbf{p}^1\Delta\mathbf{x} - \Delta c$$

The proof follows immediately from the convexity of the indirect utility function and Roy's identity. See Varian [1985] for details.

Now think of the n goods as being the same good sold in n separate markets. Assume the marginal cost of production is a constant c. Initially the goods are sold for a uniform price $\mathbf{p}_0$ but then price discrimination is allowed so that goods then sell for different prices $(p_1, \ldots, p_n)$. In this case the welfare bounds simplify to

$$(p_0 - c)\sum_{i=1}^{n}\Delta x_i \geq \Delta W \geq \sum_{i=1}^{n}(p_i - c)\Delta x_i.$$

The left-hand side of the inequality shows that a necessary condition for welfare to increase is that total output increases, a result first shown by Schmalensee [1981]. The right-hand side of the inequality shows that if the profitability of the new output exceeds that of the old output valued at the new prices then welfare must increase. This is essentially a revealed preference relation.

3 Demand for broadband

The next application involves an experiment in the 1990s that involved providing broadband internet access; for a detailed description see Edell and Varaiya [1999]. We provided broadband ISBN access to about 100 Berkeley faculty, staff and students. Every week, we changed the pricing of the service, and we could observe how users responded to various prices. We used a variety of pricing plans including flat prices, bandwidth pricing, bytes transferred, and various combinations.

Bandwidth	p128	p96	p64	p32	p16
128	**−2.0**	+.80	+.25	−.02	−.16
96	**+1.7**	**−3.1**	**+.43**	+.19	+.18
64	+.77	**+1.8**	**−2.9**	+.59	+.21
32	+.81	−1.0	**+1.0**	**−1.4**	+.15
16	+0.2	−.29	+.04	**+1.2**	**−1.3**

Table 1: Reduced form estimates. Bold fonts indicate significance at the 5% level. All own price effects are significantly negative; the cross-price effects for one-step lower bandwidths are positive.

I will only describe the bandwidth pricing here. There were 6 different levels ranging from 8 Kbs to 128 Kbs. Of course, this would hardly be considered broadband these days, but in 1998 it was the top of the line.

We varied the prices weekly and observed the user response. Using the observed responses, we estimated log-linear aggregate demand functions, with and without individual fixed effects; both specifications yielded similar elasticities so I report only the estimation without fixed effects.

The estimated demand elasticities for the 6 different bandwidths are shown in Table 1. One would expect that the demand for, say, 96Kbs bandwidth would depend negatively on its own-price and positively on the prices of other bandwidths, with the nearby bandwidths as the strongest substitutes. Indeed that is essentially what we found.

3.1 A utility-based model

This is just a reduced form, so let us examine a simple utility-based model. Assume that users get utility from the bits transferred ($u(x)$) and the time (t) it takes to transfer them. The cost of transfer time has two components: the subjective cost of time (c), which varies according to users and circumstances, and the dollar cost, which depends on the price of the chosen bandwidth ($p(b^*)$). If b^* is the chosen bandwidth for a particular consumer, revealed preference implies that

$$u(x) - [c + p(b^*)]t^* \geq u(x) - [c + p(b)]t, \tag{1}$$

for all bandwidths b.

Since bandwidth is by definition bits per unit time, we have $t = x/b$. Making this substitution and canceling the xs, we have

$$[c + p(b^*)]\frac{1}{b^*} \leq [c + p(b)]\frac{1}{b}, \tag{2}$$

for all bandwidths b. Applying some algebra yields the following expression:

$$\min_{b^*<b} \frac{p(b^*)b - p(b)b^*}{b^* - b} \geq c \geq \max_{b^*>b} \frac{p(b^*)b - p(b)b^*}{b^* - b}. \tag{3}$$

This gives us observable upper and lower bounds for c, the user's subjective cost of time.

Figure 1 depicts these bounds graphically. Define the "total cost of time" by

$$K(c) = [c + p(b)]\frac{1}{b}, \tag{4}$$

and plot these affine functions for each bandwidth b. A user with subjective time cost c will choose the bandwidth b with the lowest total cost. Conversely, an observed choice of b implies that the time cost must be bounded above and below as indicated in Figure 1. Note that a choice of the lowest available bandwidth only yields an upper bound on time cost, and a choice of the highest available bandwidth only yields a lower bound on time cost.

3.2 Distribution of the time costs

Table 2 shows the number of times that the upper and lower bounds on value of time fall in a given range. For example, 39 of the users, or about 60%, have an average upper bound on the time cost of less than 1 cent a minute,

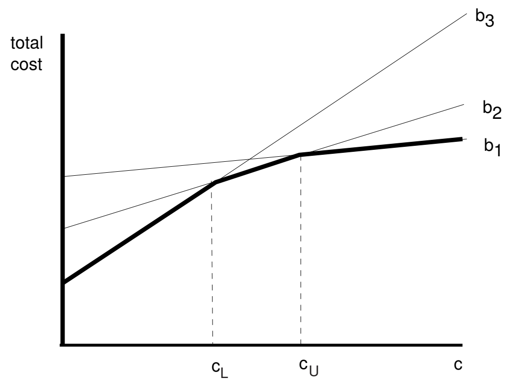

Figure 1: The straight lines are the "total cost of time" at different bandwidths. If we observe a particular bandwidth being chosen, we can calculate upper and lower bounds on the subjective time cost *c*.

Range (cents/min)	0	1	2	3	4	5	6	7	8	9	10
Count using upper bound	39	8	3	4	1	2	2	1	2	0	3
Count using lower bound	63	3	0	0	1	0	0	0	0	0	0
Count using average	47	7	2	3	3	3	1	3	1	1	0

Table 2: Count of cases where average time cost in given range is observed.

8 of the users, or about 12%, have an average upper bound greater than 1 cent a minute, but less than 2 cents a minute and so on. The last line in this table shows this count for an average of the upper and lower bounds on time cost. Figure 2 depicts the same information in a bar chart.

The remarkable thing about Table 2 and Figure 2 is the low values that users place on their time. Most of the users have a time cost of less than 1 cent a minute, with only a few users having higher time costs.

4 Auction values

The final example I want to describe is the work I did on "position auctions."

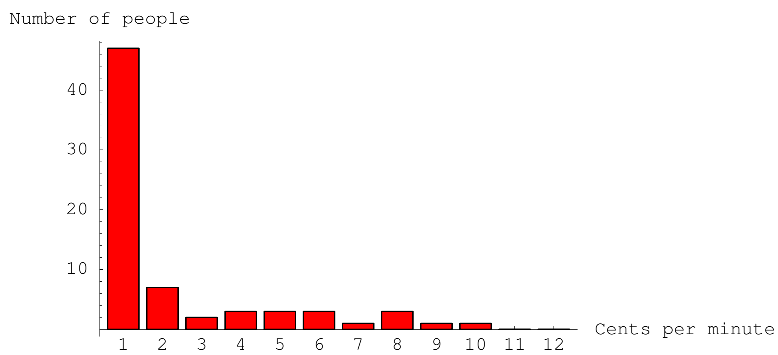

Figure 2: Histogram of the number of users with different time values.

When I went to Google in May 2002 Eric Schmidt said to me "Why don't you take a look at this ad auction, I think it might make us a little money." Eric was referring to the Google AdWords ad auction which was developed by Google employees Eric Veach and Salar Kamangar and deployed in February of 2002. In this auction, advertisers bid for position on a page of search results and pay the minimum price necessary to retain their preferred position. Levy [2009] contains a readable history of ad auctions of this sort.

I developed a game-theoretic model of the ad auction in Varian [2007] and will here describe a simplified version of that model.

Suppose that there are S slots on a page and assume that slot s will get on average x_s clicks in a given day. We number the slots from most prominent to least prominent and assume that $x_1 > x_2 > \cdots > x_S$. There is a price per click for each slot, p_s, that is determined from the bids. Finally, there is an (unobserved) value per click associated with the advertiser that appears in slot s, which we denote by v_s.

In equilibrium, each advertiser must prefer the slot it is in to any other slot which implies

$$v_s x_s - p_s x_s \geq v_s x_t - p_t x_t.$$

This is essentially a revealed preference condition similar to that described in the bandwidth example. Rearranging the condition gives us

$$v_s(x_s - x_t) \geq p_s x_s - p_t x_t,$$

which simply says that the value of the incremental (decremental) clicks in moving from s to t must be less than the incremental (decremental) cost.

It turns out that it is enough to check these inequalities for the adjacent slots. Dividing through by $x_s - x_t$ for $t = s-1, s+1$ we have

$$\frac{p_{s-1}x_{s-1} - p_s x_s}{x_{s-1} - x_s} \geq v_s \geq \frac{p_s x_s - p_{s+1}x_{s+1}}{x_s - x_{s+1}} \tag{5}$$

This gives us *observable* bounds on the unobserved value-per-click. One can also use "conversion rate" (purchases divided by clicks) to estimate a value per purchase using the identity

$$\frac{\text{value}}{\text{purchases}} = \frac{\text{value}}{\text{clicks}} \times \frac{\text{clicks}}{\text{purchases}} \tag{6}$$

The value per purchase should essentially be the profit on the item sold, so this provides a check as to whether the model is plausible. There are many other applications of this analysis, some of which are summarized in Varian [2007, 2009].

5 Summary

These are just a few examples of the power of revealed preference as an analytic construct. I'm sure it will continue to provide insights for economists for centuries to come.

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